Find all vectors that are perpendicular to the column vectors in A.

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 5 & 2 \\ 2 & 3 & 3 \end{bmatrix}$$

Nullspace

$$A^T = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 3 \\ -1 & 2 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 3 \\ 0 & 5 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & 1 \\ 0 & 5 & 5 \end{bmatrix} \rightarrow$$

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \qquad N(A^T) = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} a$$

$$\begin{bmatrix} 1 & -1 & 1 \end{bmatrix}$$

null space.

$$Ax = 0$$

$$A^T x = 0 \rightarrow (A^T x)^T = 0 \rightarrow x^T A = 0$$

left nullspace.

The nullspace is the *orthogonal* complement of the row space in $\mathbf{R}^n$.

The left nullspace is the *orthogonal* complement of the column space in $\mathbf{R}^m$.

Find all vectors that are orthogonal to the row space and the column space of A.

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 5 & 11 \\ -2 & 3 & -1 \end{bmatrix}$$

a) $\begin{bmatrix} 1 & 2 & 4 \\ 0 & -1 & -1 \\ 0 & 7 & 7 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 4 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$

$b = -c$

$a - 2c + 4c = 0$

$a = -2c$

$N(A) = \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix} a$

b) $\begin{bmatrix} 1 & 3 & -2 \\ 2 & 5 & 3 \\ 4 & 11 & -1 \end{bmatrix} = A^T$

$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & -7 \\ 0 & 1 & -7 \end{bmatrix}$ $\begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & -7 \\ 0 & 0 & 0 \end{bmatrix}$

$y = 7z$

$x + 21z - 2z = 0$

$x = -19z$

$\therefore \begin{bmatrix} -19 \\ 7 \\ 1 \end{bmatrix} a$

If **V** and **W** are orthogonal subspaces, show that the only vector they have in common is the zero vector: $\mathbf{V} \cap \mathbf{W} = \{0\}$.

assume $v \in V$ & $v \in W$ & $v \neq 0$

$v^T v \neq 0 = 0^2$

(a_1, a_2, a_3)

$a_1^2 + a_2^2 + a_3^2 = 0$

Find the orthogonal complement of the plane spanned by the vectors (1, 1, 2) and (1, 2, 3).

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$